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# WORKSHOP ON MODELS OF FRACTURE, 1–12 NOVEMBER 1999

## ABSTRACTS OF LECTURES

## (updated 4 November 1999)

The organisers wish to thank the following for their contribution to the success of this conference: European Office of Aerospace Research and Development, Air Force Office of Scientific Research, United States Air Force Scientific Laboratory
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The London Mathematical Society
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**A Newton Institute Workshop**

**Models of Fracture**

1 - 12 November 1999

Organised by Professor JR Willis

**Workshop Timetable**

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**Monday 1 November**

09.00-10.00 Registration

10.30-11.00 Coffee

11.00-12.00 KB Broberg (Dublin)
*Personal encounters with the phenomenon of dynamic crack propagation*

12.30-13.30 Lunch

14.00-15.00 LB Freund (Brown)
*Multiple neck formation in a ductile material at high strain rate*

15.00-15.30 Tea

17.00-18.00 JR Rice (Harvard)
*Problems in crack and fault dynamics*
**Newton Institute Seminar**

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**Tuesday 2 November**

10.30-11.00 Coffee

11.00-12.00 F Lund (Chile)
*Elastic theory of dynamic cracks*

12.30-13.30 Lunch

14.00-15.00 Y Brechet (LTPCM)
*Fracture of industrial alloys: micromechanics and macroscopic anisotropy*

15.00-15.30 Tea

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**Wednesday 3 November**

10.30-11.00 Coffee

11.00-12.00 M Adda-Bedia (ENS)
*Quasistatic and dynamic stability of straight crack propagation*

12.30-13.30 Lunch

14.00-15.00 H Nakanishi (Kyushu)
*Dynamical fracture in continuum model*

15.00-15.30 Tea

16.00-17.00 J-B Leblond (Paris)
*Crack kinking from an initially closed crack - cases of an ordinary crack, and an interface one, with and without friction*

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**Thursday 4 November**

10.30-11.00 Coffee

11.00-12.00 G Dal Maso (Trieste)
*A variational formulation of softening phenomena in fracture mechanics*

12.30-13.30 Lunch

14.00-15.00 GA Francfort (Paris)
*Revisiting brittle fracture from an energy minimization standpoint*

15.00-15.30 Tea

16.00-17.00 M Falk (Harvard)
*The plastic dynamics of brittle vs ductile fracture in amorphous solids*

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**Friday 5 November**

10.30-11.00 Coffee

11.00-12.00 J Sivaloganathan (Bath)
*On the existence and location of singularities arising in variational problems of nonlinear elasticity*

12.30-13.30 Lunch

14.00-15.00 A Braides (Rome)
*Non-convex discrete systems and fracture*

15.00-15.30 Tea

16.00-17.00 G Buttazzo (Pisa)
*Elastic structures modelled by measures and applications to shape optimization*

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**Monday 8 November**

10.30-11.00 Coffee

11.00-12.00 E Sharon (Hebrew)
*Experiments on dynamic fracture in brittle amorphous materials: on the validity of the LEFM theory of dynamic fracture ........*

12.30-13.30 Lunch

14.00-15.00 E Ching (Hong Kong)
*Stability analysis of fracture propagation in cohesive zone models: a summary with emphasis on difficulties*

15.00-15.30 Tea

16.00-17.00 V Kovtunenko (Stuttgart)
*Sensitivity of cracks in 2D-elastic solids*

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**Tuesday 9 November**

10.30-11.00 Coffee

11.00-12.00 L Truskinovsky (Minnesota)
*Bi-modal surface energy and microcracking*

12.30-13.30 Lunch

14.00-15.00 K Ranjith (Harvard)
*Slip dynamics at a dissimilar material interface*

15.00-15.30 Tea

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**Wednesday 10 November**

10.30-11.00 Coffee

11.00-12.00 LI Slepyan (Tel Aviv)
*Phenomena in fracture revealed by lattice models*

12.30-13.30 Lunch

14.00-15.00 WJ Drugan (Wisconsin)
*Analytical mechanics-based modeling of dynamic fragmentation in brittle materials*

15.00-15.30 Tea

16.30-17.30 JR Rice (Harvard)
*Mode of rupture propagation on faults: Expanding cracks versus self-healing slip pulses*
**Seminar at Bullard Laboratory**

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**Thursday 11 November**

10.30-11.00 Coffee

11.00-12.00 AB Movchan (Liverpool)
*A pertubation model for a three-dimensional crack on an interface*

12.30-13.30 Lunch

14.00-15.00 R Craster (Imperial)
*Applications of invariant integrals*

15.00-15.30 Tea

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**Friday 12 November**

10.30-11.00 Coffee

11.00-12.00 JR Willis (Cambridge)
*Dynamic perturbation of a crack propagating in a viscoelastic medium*

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# Quasistatic and dynamic stability of straight crack propagation

*M. Adda–Bedia*
*Paris*

Recent experiments have shown that it is possible to control crack propagation at very low speeds. This is done by means of internal diffusion fields which generate in the material inhomogeneous stress fields. Two experiments of this kind will be taken as an illustration. The first one concerns a crack travelling in a glass strip submitted to a non-uniform thermal field. The second experiment concerns multi-crack propagation induced by the drying of colloidal suspensions. Both of them show that cracks undergo numerous instabilities that depend of the geometry of the experiment and of the control parameters of the diffusion field. A theoretical analysis of the morphological instability mechanisms of a single crack and multiple crack propagation will be presented. However, daily crack propagation often occurs at very high velocities. A still unsolved question concerns the onset of branching instability. In 1970, Eshelby proposed a simple branching mechanism based on the fact that a crack branches in order to decrease the breaking speed. In this second part, we will review this question and compute exactly the branching velocity threshold and the branching angle in the light of Eshelby's hypothesis.

**References**

M. Adda–Bedia and Y. Pomeau : *Crack instabilities of a heated glass strip*, Phys. Rev. E **52** (1995) 4105-4113.

M. Adda–Bedia and M. Ben Amar : *Stability of quasi-equilibrium cracks under Mode I loading*, Phys. Rev. Lett. **76** (1996) 1497-1500.

M. Adda–Bedia, M. Ben Amar and Y. Pomeau : *Morphological instabilities of dynamic fractures in brittle solids*, Phys. Rev. E **54** (1996)

# Non-convex discrete systems and fracture

*Andrea Braides*
*SISSA, via Beirut 4, 34013 Trieste, Italy* Email: braides@sissa.it

The use of non-convex discrete energies to derive a continuum theory by using a variational approach was recently considered by Truskinovsky [7] and by Braides, Dal Maso and Garroni [12]. In both those papers the authors consider an array of material points linked by nearest-neighbour interactions and an energy of the form

$$E_\varepsilon(u) = \sum_i \varepsilon \psi_\varepsilon\Big(\frac{u_i - u_{i-1}}{\varepsilon}\Big), \tag{1}$$

where $\varepsilon$ is the distance between neighbouring points in the reference configuration, $u_i$ denotes the position of the $i$-th material point ($i = 1, \dots, L/\varepsilon$) in the deformed configuration and $\psi_\varepsilon$ is a Lennard Jones type potential characterized by being convex until a threshold $T_\varepsilon$ and then concave. In both cases, though with some technical differences, the authors highlight that the effect of the convex part is to give rise to a bulk (elastic) energy, while the concave part contributes to a penalization of fracture. Minimizers, local minimizers and stationary points under suitable boundary conditions and body forces are characterized as having only a finite number of interactions lying in the concave region of $\psi_\varepsilon$ which give rise to fracture in the limit description, while all the other interactions contribute to the bulk. The limit continuum energy can be explicitly described by using the terminology of $\Gamma$-convergence (which is a variational convergence that ensures the convergence of minimum problems, see [11] or [8]), and the theory of free-discontinuity functionals, of the form

$$\mathcal{E}(u) = \int_{(0,L)\setminus S(u)} f(\dot u)\, dt + \sum_{t\in S(u)} g([u]), \tag{2}$$

defined on piecewise-Sobolev functions on the segment $(0, L)$ ($S(u)$ denotes the set of discontinuities of $u$ and $[u]$ stands for the jump of $u$), and can be carried to dimension higher than one by using the framework of special functions of bounded variation (see [4]). In the case treated in [7] the limit energy densities $f$ and $g$ are defined as the limit of the convex part of $\psi_\varepsilon$ and of a suitable scaling of its concave part, respectively. If the convex/concave structure of $\psi_\varepsilon$ is missing a more delicate analysis must be performed (see [10]) This approach can be pushed further to deal with scalar-valued functionals of the same form, defined on higher-dimensional lattices. The fundamental technique is a 'slicing approach' which allows to reduce to 1-dimensional sections, and hence to the theory outlined above. Long-range interactions (both in a one dimensional and in a multi-dimensional setting) can be analyzed using a superposition procedure in some situations, when the effect of the interactions decay sufficiently fast with respect to the distance in the reference lattice (see [9]). If this fast-decay assumption fails then the limit may contain also a non-local term, and the surface energy is affected by a boundary-layer effect (see [6]). The methods illustrated above must be integrated by additional arguments in the case of vector-valued functions. Even in the simplest case of functionals related to Griffith's theory of brittle fracture,

non-central interactions must be added, with non-trivial technical problems due to the non-applicability of a 'slicing approach' (see [4]).

**References**

[1] R. Alicandro, M. Focardi and M.S. Gelli, Finite-difference approximation of energies in fracture mechanics, Preprint SISSA, Trieste, 1999.

[2] L. Ambrosio, Existence theory for a new class of variational problems, *Arch. Rational Mech. Anal.* **111** (1990), 291–322.

[3] L. Ambrosio and A. Braides. Energies in SBV and variational models in fracture mechanics. In *Homogenization and Applications to Material Sciences*, (D. Cioranescu, A. Damlamian, P. Donato eds.), GAKUTO, Gakkōtosho, Tokio, Japan, 1997, pp. 1–22.

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[7] A. Braides, G. Dal Maso and A. Garroni, Variational formulation of softening phenomena in fracture mechanics: the one-dimensional case, *Arch. Rational Mech. Anal.* (1999), to appear.

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[10] A. Braides and M.S. Gelli, Limits of discrete systems without convexity hypotheses. Preprint SISSA, 1999.

[11] G. Dal Maso, *An Introduction to* $\Gamma$*-convergence*, Birkhäuser, Boston, 1993.

[12] L. Truskinovsky, Fracture as a phase transition, *Contemporary research in the mechanics and mathematics of materials* (R.C. Batra and M.F. Beatty eds.) CIMNE, Barcelona, 1996, 322–332.

# Mechanisms of Fracture in Industrial alloys

*Yves Bréchet*
*INPG Grenoble*

Industrial alloys present a wide variety of fracture mechanisms, depending on the microstruture and on the mode of loading. Recent development in micromechanics allow to model in certain cases both the fracture characteristics and the anisotropy of these characteristics depending on the microstrutural features such as coarse precipitates, texture, or grain size. The present seminar aims at illustrating by a number of examples these approaches linking mechanics and metallurgy: Fracture of aeronautical Aluminium alloys in tensile loading and coupling with anisotropy Fracture of aeronautical Aluminium alloys in fatigue loading Fracture of duplex stainless steels The seminar will outline *in fine* some open problems in materials science for which coupling between physical metallurgy of fracture and micromechanics approach are badly needed, in particular as far as the defects occurring from the processing stage are concerned .

# Personal encounters with the phenomenon of dynamic crack propagation

*K B Broberg*
*Dublin*

My interest in dynamic crack propagation dates back to the mid 1950's when I studied scabbing of solids under explosive attack. Several very accurate experimental results on dynamic crack propagation existed at that time, notably those associated with the German school (e.g. Schardin 1950). They indicated a material dependent terminal crack velocity, surface roughness increasing with the crack velocity, and branching at high velocities. It was puzzling that a) the terminal velocity was considerably lower than the Rayleigh wave velocity, b) the surface roughness increased rather than decreased (in the light of experiences of embrittlement at high loading velocities), and c) branching could occur without appreciable loss of crack edge velocity. Contrary to experimental results, a solution for an expanding crack (Broberg 1960) indicated the Rayleigh wave velocity as terminal. So did a determination of the energy flux into the crack edge (Broberg 1964), but I assumed that the local, considerably reduced, Rayleigh wave velocity in the highly strained region at the crack edge set the limit, a hypothesis which, however, proved to be erroneous (Broberg 1973). At the Lehigh symposium 1972, a paper was presented showing an astonishingly steep rise of the energy dissipation at high crack velocities in PMMA (Paxson and Lucas 1973). This gave a clue to answering questions a) and c). As for the implications on branching, Pärletun (1979) showed that branching is postponed far beyond the point when it is energetically possible and that branching may indeed occur without noticeable reduction of the crack velocity. Questions a) and b) seemed to be answered by the cell model (Broberg 1979), but it implied the loss of a material length parameter, which made it impossible to maintain a unique relation between energy flux and velocity. However, this appeared to be a reality, as confirmed experimentally by Ravi-Chandar (1982). Johnson (1992) used the cell model for numerical simulations, which could reproduce most of the experimental results, including branching. Later, I got interested in mode II cracks, which may run at intersonic velocities. This has recently been confirmed by experiments (Rosakis *et al.*1999).

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Schardin, H., 1950. *Glastechnische Berichte*, **23**, 1-10; 67-79; 325-336.

# Elastic structures modelled by measures and applications to shape optimization

*Giuseppe Buttazzo*
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*Via Buonarroti, 2*
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The point of view we adopted is that an (hyper)elastic structure is identified once its stored energy functional is given. For instance, if $\Omega$ is a regular domain, the energy

$$E(u) = \int_\Omega \left(\frac{\lambda}{2}(\operatorname{div} u)^2 + \mu|e(u)|^2\right) dx$$

identifies, in the framework of linear elasticity, the elastic homogeneous isotropic body $\Omega$. The limit of this approach appears when one has to deal with complex structures, where pieces of different dimensions may be joined together, and in problems of shape optimization, where the structure is not prescribed but represents the unknown of the problem. In particular, in many cases of this latter type, it is known that a classical solution does not exist and the optimal structure has a meaning only in a suitable "relaxed" sense. The approach we present consists in identifying an elastic structure to a nonnegative measure $\mu$; the stored energy functional then takes the form

$$E(u) = \int \left(\frac{\lambda}{2}(\operatorname{div} u)^2 + \mu|e(u)|^2\right) d\mu$$

and admissible displacements $u$ then belong to a Sobolev space $H^1_\mu$ which can be suitably defined. In this way structures of any kind of complexity can be treated, and several shape optimization problems admit an optimal (measure) solution.

**References**

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[2] G. BOUCHITTE, G. BUTTAZZO, P. SEPPECHER: *Shape optimization solutions via Monge-Kantorovich equation.* C. R. Acad. Sci. Paris, **324-I** (1997), 1185–1191.
[3] G. BOUCHITTE, G. BUTTAZZO: *Characterization of optimal shapes and masses through Monge-Kantorovich equation.* Paper in preparation.

## Stability analysis of fracture propagation in cohesive zone models: a summary with emphasis on difficulties

*E. Ching*
*Hong Kong*

In this talk, I shall give a summary of our investigation of the dynamic stability of fracture propagation in cohesive-zone models and especially emphasize on how these models might be mathematically ill-posed for stabilty analysis. In our approach, we analyse the linear stability by evaluating the first-order response of the crack to a small spatially oscillating stress. I shall first illustrate our method using a simple one-dimensional model [Ching, Langer, and Nakanishi, Phys. Rev. E **52**, 4414 (1995)]. In one dimension, the linear response is a small oscillation in the velocity of the crack tip. For this simple model, the linear response coefficient can be computed explicitly, and all its poles are found to be stable exponentially decaying modes, as expected. However when this analysis is applied to mode-I fracture in a two-dimensional model [Ching, Langer, Nakanishi, Phys. Rev. E **53**, 2864 (1996)], it turns out there is no unique solution [Langer and Lobkovsky, J. Mech. Phys. Solids **46**, 1521 (1998)]. I finally present calucations for mode-III fracture in a two-dimensional model with friction. We shall see that a mathematical inconsistency is explicitly found in the stability analysis despite that the model gives steady-state results that are physically sound.

## Applications of invariant integrals

*R. V. Craster*
*Imperial College, London SW7 2BZ*

This talk will describe applications of invariant integrals based upon ideas due to Atkinson (1975), Eshelby (1970), Nilsson (1973), Rice (1968), and others to fracture problems in poroelastic, thermoelastic or non-homogeneous or layered dynamic elastic materials. An iterative approach to solving some crack problems based upon generalized ray theory will also be discussed, together with an application of weight functions.

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# A variational formulation of softening phenomena in fracture mechanics

*Gianni Dal Maso*
*SISSA, Trieste, Italy*

Discrete models of particles subject to nearest-neighbour non-linear interactions are used to approximate continua allowing for softening and fracture. The qualitative properties of all continuous one-dimensional variational models obtained in this way are examined. A detailed study is carried out of local minima and stationary points for the continuous models. Scale effects are discussed. These results are proved in the reference given below.

## Reference

Braides A., Dal Maso G., Garroni A.: Variational formulation of softening phenomena in fracture mechanics: the one-dimensional case. *Arch. Rational Mech. Anal.* **146** (1999), 23-58.

# Analytical Mechanics–Based Modeling of Dynamic Fragmentation in Brittle Materials

*W. J. Drugan*
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The dynamic fragmentation of brittle materials is an extremely complex process involving the nucleation and propagation of myriad microcracks that finally coalesce, breaking the solid into fragments. To date, the most widely applied theoretical models of the process involve some type of relatively simple global energy balance argument to predict fragment sizes and velocities. Very recently, computational modeling of the phenomenon has been carried out by large finite element calculations based on the incorporation of cohesive surfaces between standard elastic elements, to serve as prospective fracture paths in a dynamic simulation. In the present study we propose and analyze a simple analytical mechanics-based model of dynamic fragmentation which, rather than employing a global energy balance, adopts a cohesive surface formulation and analyzes the time-varying dynamic deformation of a prospective brittle fragment. This leads to predictions of time to initiate fragmentation and minimum fragment size, as a function of material properties and the applied strain rate. These predictions are compared with the energy-based models of Grady (1982) and Glenn and Chudnovsky (1986), and with the recent numerical finite element simulations of Miller, Freund and Needleman (1999). In an interesting confirmation of all the models, our predictions and those of the two energy-based models converge at extremely high strain rates, the regime in which one would expect the energy models to be valid. However, our new model implies that the energy-based models' regime of validity is restricted to surprisingly (and perhaps unphysically) high strain rates.

**References**

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# The plastic dynamics of brittle vs ductile fracture in amorphous solids

*Michael L. Falk*
*Division of Engineering and Applied Sciences*
*Harvard University*

The microscopic processes that underlie plasticity in crystalline solids are relatively well understood in terms of dislocations. Nonetheless, making a connection between these dislocation dynamics and theories of macroscopic plasticity in order to understand phenomena such as the brittle/ductile transition in fracture remains elusive. Meanwhile, the micromechanics that give rise to a nearly identical range of phenomena in noncrystalline materials are relatively poorly understood. This route to understanding plasticity may prove simpler and could shed light on phenomena in both types of solids. Molecular dynamics investigations of fracture in model noncrystalline systems reveal that the relative ductility, ie fracture toughness, is sensitive to the particulars of the interatomic potential. Similar investigations of the inelastic shear response of this model system reveal analogous time and history dependent behavior to that seen in actual materials. Examination of the rearrangements underlying these phenomena leads to the conclusion that the basic unit of inelastic shear can be understood as a microscopic two-state system. A theory of the dynamics of inelastic deformation under shear is developed from this assumption and compared to the simulation results. The implications for fracture are discussed.

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## Revisiting brittle fracture from an energy minimization standpoint

*Gilles Francfort*
*L.P.M.T.M.,*
*Université Paris-Nord, 93430 Villetaneuse, France* [Email francfor@lpmtm.univ-paris13.fr]

I will report on the result of a collaborative effort with B. BOURDIN (Danmarks Tekniske Universitet, Lyngby), A. CHAMBOLLE (Université Paris–Dauphine, Paris), I. FONSECA (Carnegie Mellon University, Pittsburgh) and J.J. MARIGO (Université Paris–Nord, Paris). In the first part of the talk, I will briefly recall the basic principles of classical quasistatic brittle fracture and evoke the difficulties that the theory encounters when attempting to predict the initiation, path or growth of a crack. I will then propose a "slight" deviation of the model which alleviates the abovementioned difficulties at the expense of a postulate of strong stability of the equilibrium configurations throughout the (quasistatic) evolution. A second part of the talk will be devoted to the mathematical treatment of the proposed model; this will be rather brief because I expect that it will be covered in more details in other talks, most likely those of G. DAL MASO or A. BRAIDES. In the third part of the talk I will demonstrate how the resulting formulation is amenable to efficient numerical algorithms that rely on variational approximation techniques. I will finally present three numerical crack evolutions in settings that are, to my knowledge, beyond the predictive range of the classical methods. Background material for the talk may be found in the following articles and references therein.

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G. A. FRANCFORT, J.J. MARIGO. Revisiting brittle fracture as an energy minimization problem. *J. Mech. Phys. Solids*, **46**–8, 1998, 1319–1342. B. BOURDIN, G. A. FRANCFORT, J.J. MARIGO. Numerical experiments in revisited brittle fracture: . to appear in *J. Mech. Phys. Solids.*

# Multiple Neck Formation in a Ductile Material at High Strain Rate

*L. B. Freund*
*Brown University*

When a ductile metal cylinder is expanded at a slow rate, say by internal pressure, it deforms more or less uniformly into the plastic range until a strain localization, or "neck", develops at some point on the circumference; ductile fracture follows almost immediately at this site. In contrast, if the same plastic deformation is induced at a high rate of loading, experiments reveal that many necks form around the circumference of the cylinder at more or less equally spaced sites. The onset of this phenomenon and its sensitivity to material parameters are discussed on the basis of numerical simulations of the deformation process and a dynamic bifurcation analysis. It is shown that formation of multiple necks which grow at rates large compared to the rate of background deformation occurs naturally in the process. Material strain rate sensitivity, which is important in describing the growth of necks into ductile fractures, does not seem to play a significant role in their formation.

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## Sensitivity of cracks in 2D-elastic solids

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Research of crack problems has the principal difficulty due to the non-regular boundary, which leads to the loss of the whole solution smoothness. The main subject of our interest concerns the shape sensitivity analysis in these problems. This means that we investigate variations of its solution, energy functional, or stress intensity factors, to perturbations of a crack. The reason is that the crack propagation depends on the derivatives of energy functional by the Griffith criterion, or on the stress intensity factors by the Irwin criterion. The admissible variations with respect to the crack perturbations are shown with stress on the mathematical difficulties.

We consider the static model of classical linear elasticity for a two-dimensional solid with a crack. First, there are assumed different types of boundary conditions imposed on the crack faces, such as the Neumann, Signorini and friction conditions. Second, the crack length and shape are both under the perturbation for the rectilinear and curvilinear crack forms.

We can obtain full expansion in the linear models with stress-free boundary conditions at a crack. This allows us to formulate a condition of the locally optimal crack from the variational principles. In nonlinear crack problems restricted by unilateral constraints imposed at a crack, we have only the first-order variations and can formulate a condition of the crack stationarity or growth by the Griffith criterion.

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## Crack kinking from an initially closed crack — cases of an ordinary crack and an interface one, with and without friction

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Crack kinking in elastic solids in plane strain situations is studied in the case where the crack is initially closed but kinking opens it. One considers first the case of an ordinary (not interface) crack. The first problem which arises is to determine the form of the asymptotic stress field near the crack tip prior to kinking. It has been solved notably by Deng, who showed that because of contact between the crack lips, the classical singular mode I field is replaced by another, bounded one involving a new non-singular stress. In the absence of friction, this non-singular stress just represents a uniform compression perpendicular to the crack. If friction is present, it also involves some uniform shear stress. From there, and using an argument of positive homogeneity of degree 1 of the type of problems considered plus some change of scale, one derives universal formulae for the first two terms of the expansion of the stress intensity factors at the tip of the open, extended crack in powers of the crack extension length. It is remarkable that even in the presence of friction, no "history effects" appear in these formulae in spite of the fact that the problem is then basically of incremental character. The various universal functions of the kink angle and the friction coefficient involved in these formulae are identified through finite element computations (in elasticity with unilateral contact, with or without friction). From there, and using Goldstein and Salganik's "principle of local symmetry", one can determine the kink angle, which is found to always precisely amount to 77.3 degrees, irrespective of the friction coefficient. This result is confirmed by recent experiments of Pinna which have evidenced a kink angle of the order of 70 degrees. In contrast, the initial curvature parameter of the crack extension is found to depend upon both non-singular stresses of the initial crack. The problem of whether or not, after the initial kink, the crack tends to come back to its original direction is finally investigated, and it is found that this depends upon the respective values of the two initial non-singular stresses. One considers then the case of an interface crack, which is in fact always asymptotically closed, whatever the loading applied. The question of the form of the asymptotic stress field near the crack tip has again been solved by some previous author, namely Comninou (with or without friction). Again, in the singular, dominant term, the "mode I" contribution disappears so that this term only depends upon a single stress intensity factor, of "mode II". Attention is paid here only to the first term of the expansion of the stress intensity factors at the extended crack tip in powers of the crack extension length, i.e. to the value of these stress intensity factors just after the kink. Using the same type of arguments as for an ordinary crack, one derives the expression of these stress intensity factors. Again, no "history effects" appear in this expression. The values of the universal functions of the kink angle, the friction coefficient and the mismatch of elastic properties between the materials which appear here are again determined by finite element computations. Also, application of the principle of local symmetry yields the value of the kink angle as before. This angle depends upon both the mismatch of elastic properties and the friction coeffi-

cient, but only weakly upon the first argument.

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# Elastic theory of dynamic cracks, with an emphasis on the fracture of thin plates

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Recent experiments on the fracture of thin samples of glass and plexiglass have revealed unstable crack dynamic behaviour. These experiments have taken a close quantitative look at the crack-tip velocity fluctuations, ultrasound emissions, and surface heterogneities associated with long standing puzzles in the tensile cracking of very brittle materials. When the motion of an elastic singularity deviates from a straight line, energy balance considerations are insufficient to determine the singularity's motion since they provide only one equation to determine the dynamics of two, or three, degrees of freedom, depending on the dimensionality of the system. Thus, in addition, the equations of (linear) momentum balance must be considered. In so doing, there results a new elastic force that does no work and that consequently cannot be obtained on the basis of energy considerations alone. The way to go about showing the existence of this no-work-performing force is to write the equations of dynamic elasticity in energy-momentum conservation form and to integrate them out within a volume that excludes any singularities, in a manner that is standard for quasi-static processes. In order to understand how this force affects crack dynamics, it is convenient to think of the crack tip as a superposition of many infinitesimal dislocations. For a single dislocation, it is possible to establish the exact form of this force: it is perpendicular to the direction of motion, it depends linearly on the dislocation's velocity so that it vanishes when the dislocation is at rest, and it is present only for dynamic loading. These facts suggest a possible scenario to understand the dynamic instability of fast cracks in thin brittle materials in terms of a resonant interaction between crack tip and Rayleigh-Lamb modes, and has motivated a detailed study of said interaction.

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## A perturbation model for a three-dimensional crack on an interface

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This talk includes the study of the weight functions for three-dimensional interfacial cracks and the perturbation analysis associated with a small deviation of the crack front; the unperturbed crack occupies a half-plane on the interface between two half-spaces. We begin with the scalar problem and consider both static and dynamic cases. Then, for the static problem of linear elasticity we discuss the structure of the weight functions and asymptotics for the stress-intensity factors for an interfacial crack with a wavy front. Here, we use the integral equation analysis developed in [1]; the symmetric weight functions for a 3D static interfacial crack have been constructed in [2]; the asymptotic algorithm related to small perturbations of the crack front is based on the results of [3-6]. [1] J.R. Willis, Fracture mechanics of interfacial cracks. J. Mech. Phys. Solids 19 (1971), 353–368.
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# Dynamical Fracture in Continuum Models

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A mathematical technique to solve a simple continuum model of fracture dynamics is reviewed. The model is defined as a dynamical one in the sense that the dynamical features of the fracture propagation can be calculated from a given external physical condition. I will first present the simplest possible case to illustrate the technique, and then extend it to the case with the cohesive zone, which suppresses the stress divergence, and the case with the surface dissipation. It will be shown that a pulse solution with a self-healing fracture can be also obtained. Finally, it is pointed out that the straight crack propagation in an isotropic medium is not stable, which will be discussed by E. Ching in detail.

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# Slip dynamics at a dissimilar material interface

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It has been shown recently [Renardy (1992), Adams (1995), Simões and Martins (1998)] that steady frictional sliding along an interface between dissimilar elastic solids with Coulomb friction acting at the interface is ill-posed for a wide range of material parameters and friction coefficients. The ill-posedness is manifest in the unstable growth of interfacial disturbances of all wavelengths, with growth rate inversely proportional to the wavelength. We first establish the connection between the ill-posedness and the existence of a certain interfacial wave in frictionless contact, the generalized Rayleigh wave of Weertman (1963) and Achenbach and Epstein (1967). Precisely, it is shown that for material combinations where the generalized Rayleigh wave exists, steady sliding with Coulomb friction is ill-posed for arbitrarily small values of friction. Secondly, regularization of the problem by a friction law motivated by the experiments of Prakash and Clifton (1993) is studied. We show that a friction law with no instantaneous dependence on normal stress but a simple fading memory of prior history of normal stress makes the problem well-posed.

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# Problems in crack and fault dynamics[1]

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Recent observations on the dynamics of crack and fault rupture are described, together with related theory and simulations in the framework of continuum elastodynamics. Topics include configurational instabilities of tensile crack fronts (crack front waves, disordering, side-branching), the connection between frictional slip laws and modes of rupture propagation in earth faulting, especially conditions for formation of self-healing slip pulses, and the rich faulting and cracking phenomena which result along dissimilar material interfaces due to coupling between slippage and normal stress alteration. Studies of the unsteady dynamic propagation of tensile cracks in 3D solids have led to discovery of a new type of elastic wave which propagates laterally along the front of a growing crack. The result has implications for the disordering of initially smooth fracture propagation by small heterogeneities in fracture energy. [1-5] Observations suggest that a side-branching instability of crack path is responsible for the limiting speed of tensile ruptures. Attempts to explain that theoretically are reviewed. [6-11] For the dynamic propagation of slip rupture along earthquake faults, frictional interaction between the sliding surfaces is critical to determining the character of the process. Progress has been made on laboratory-motivated constitutive descriptions of sliding which are rich enough to predict when slip is stable or unstable. In addition, there is new understanding of dynamical interactions between friction and rupture propagation which explains when ruptures occur in the form of a classical enlarging shear crack, or in the form of a self-healing pulse in which slip arrests shortly after passage of the rupture front. Slip inversions for large earthquakes favor the latter mode. [12-14] Extremely rich behavior occurs for dynamic problems of slip and cracking along dissimilar material interfaces, in which case there is strong coupling between slippage and normal stress alteration. During cracking, zones of contact and frictional sliding have been observed at the crack tip and are predicted in elastodynamic simulations. The problem of steady sliding under classical Coulomb friction is unstable and actually ill-posed; regularizations have been devised based on experimentally motivated modifications of the friction law for situations of rapidly varying normal stress, and these lead to self-healing pulses of slip propagation. [15-21] Cracking along dissimilar material interfaces is similarly complex, with the opening part of the fracture preceded by a zone of sliding contact at the rupture tip, and with intersonic rupture propagation speeds. Such features, seen in experiments, are also reproduced in theoretical calculations [22-24].

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# Mode of rupture propagation on faults: Expanding cracks versus self-healing slip pulses[1]

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Frictional interaction between sliding fault surfaces is important to determining the character of the earthquake process. Depending on the form of that interaction, rupture could be expected to occur in the form of a classical enlarging shear crack, or in the form of a self-healing pulse in which slip arrests shortly after passage of the rupture front. Seismic inversions for large earthquakes suggest that nature prefers the self-healing mode [1], and that has induced vigorous research on the way that such a mode could develop. The issue is important not only for the character of strong ground motions, but also because features controlling the mode of rupture are thought to be an important to understanding the spatio-temporal complexities of earthquake sequences and the low overall driving stress at which some major faults are inferred to operate. It has been shown that the self-healing mode can be generated on velocity-weakening faults, described within the rate- and state-dependent friction formulation, when the constitutive law allows for restrengthening in the absence of continuing slip [2]. Further, it has recently been shown that the self-healing mode is favored, relative to the expanding crack mode, on faults which are subject to relatively low applied shear stress (outside of some localized high-stress, or low-strength, zone where the rupture nucleates) [4]. Specifically, a critical level of applied stress (outside the nucleation zone) has been identified below which no indefinitely expanding rupture of the crack-like mode can exist. Simulations support the conclusion that all ruptures propagating below that stress level are of the self-healing type, and further provide guidelines, based on a dimensionless measure of velocity weakening at typical dynamic slip rates, for when rupture at higher stress levels will be in the self-healing or the crack-like mode. The crack-like mode occurs when the measure is small. Self-healing pulses have also been shown to occur under broad conditions when elastically dissimilar materials slide [5], a possibility suggested long ago [6]. Indeed, it has recently been learned (see abstact by K. Ranjith and J. R. Rice) that the sliding of elastically dissimilar materials, even under a constant coefficient of friction, is dynamically unstable to small perturbations (and generally ill posed) under a remarkably wide parameter range, including realistic values of the friction coefficient. Some steady traveling pulse solutions have been identified [7] in which rupture propagates along an interface of only slightly dissimilar materials (e.g., shear wave speeds differ by less than 30 to 40%); the pulse travels at a generalized Rayleigh wave speed, and rupture occurs not by the concentration of shear stress at the rupture front, but

[1]Seminar to be given at Bullard Labs, Department of Earth Science, Cambridge, UK, 10 November 1999 (also as part of theme on models of fracture at Isaac Newton Institute, Cambridge, program on Mathematical Developments in Solid Mechanics and Materials Science )

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rather by the reduction of normal compressive stress to whatever level suffices to allow slip under the remotely applied shear stress. Further, the self-healing rupture mode can be generated by wave effects in rupture along faults of highly heterogeneous strength and fracture properties, effectively containing local barriers to rupture [8-10].

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# Experiments on dynamic fracture in brittle amorphous materials: on the validity of the LEFM theory of dynamic fracture and the existence of crack front waves

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We report on the results of experiments on Mode 1 fracture in brittle amorphous materials (PMMA and soda lime glass). We focus on the dynamic stages of the fracture process with an aim to verify the predictions of linear elasticity fracture mechanics (LEFM). We show that up to a velocity $v_c = 0.4V_R$ (where $V_R$ is the Rayleigh wave speed) a single crack exists and propagates according to the equation of motion of a single straight crack given by LEFM [1]. Beyond that velocity the crack becomes unstable, a repetitive process of micro branching accompanied by increasing oscillations in the crack velocity occurs [2]. At this stage the equation of motion is no longer a valid description of the mean velocity of the crack as the single crack condition that leads to the equation is violated. The validity of the theory is further demonstrated beyond $v_c$ by concentrating solely on instantaneous "single crack" states that can occur between successive branching events. At those instants, which correspond to the highest peaks in the crack velocity measurements, all of the energy flux released by the fracture process flows into a single crack. As the theory predicts that a crack has no inertia, it can attain immediately its equilibrium velocity. The measurements show, that indeed, the highest peaks in the crack velocity coincide with the theoretical single crack velocity curve at all velocities up to $0.9V_R$ [3]. Another prediction of LEFM for the case of a Mode 1 crack propagating in a three-dimensional body, concerns the stability of the crack front to perturbations. Recent analytical calculations [4] and computer simulations [5] found that a perturbation of a straight crack front in a homogeneous material, develops into two counter- propagating front waves which propagate along the running crack front. The predicted velocity of the waves is $0.94V_R$ with respect to their source. Their decay rate depends on the derivative of the fracture energy of the material with respect to the crack velocity. For a material with a constant fracture energy the decay rate is predicted to be zero. In experiments in soda lime glass we observe these front waves and confirm some of the above theoretical predictions. A propagation velocity of $V_R \pm 5\%$ was measured over a wide range of mean crack velocities. The decay rate of the waves in glass (which has a nearly constant fracture energy) is extremely low, whereas it is much higher in PMMA (where the fracture energy increases with velocity). In addition, we observe a linear superposition between interacting front waves and measure their (very high) reflection coefficient from the plates surface. It is shown that in the case of glass, the micro-branches that appear beyond $v_c$ act as the main source of the front waves.

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## On the existence and location of singularities arising in variational problems of nonlinear elasticity

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In this talk we consider the variational problem of determining the minimum energy configuration of a nonlinear elastic body under prescribed boundary displacements. Work in [1] shows that minimisers may develop point discontinuities corresponding to cavities forming in the deformed body (see also [2] and the significant extension in [3]). We present an isoperimetric inequality for (discontinuous) deformations which yields bounds on the boundary data for which discontinuous minimisers exist (see [4]). Using an invertibility condition introduced in [3] we show the existence of minimisers with potential discontinuities at any finite set of prescribed points (see [5]). Finally, we describe an approach which predicts the energetically optimum location at which a singularity will initiate (see [6]). This is joint work with S.J. Spector (Southern Illinois) and in part with S. Muller (Max-Planck Institute, Leipzig).

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# Phenomena in fracture revealed by lattice models

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Square- and triangular-cell lattices, consisting of point particles connected by massless elastic or viscoelastic bonds, are considered. These models of a structured material allow the following main phenomena in crack propagation to be revealed: *Structure- and crack-speed-dependent radiation* of high-frequency waves excited by the propagating crack front. (It was first found by Slepyan, 1981a; a survey of the main following works in this topic was represented in Slepyan, 1998.) The radiation intensity does not disappear in zero crack speed limit, has a global minimum, roughly, at a half critical speed and tends to infinity when the speed approaches the critical value. It creates wave resistance to the crack propagation as a positive difference between the global (macro-level) and local (micro-level) energy release rate. (The global rate is given by a long-wave/low-frequency asymptotic approximation of the lattice solution.) Also, the radiation can lead to damage of the crack surfaces (Marder and Gross, 1995) thus increasing the resistance. *Supersonic crack propagation* is not forbidden in the lattices. In this case, the far-field energy flux does not exist, and the crack takes energy from neighboring initially stressed layers or from a high-frequency wave with an energy flux toward the crack (Slepyan, 1981b). *Structure-associated dynamic amplification factor* appears to be a governing phenomenon in crack propagation (Slepyan, 1999). In a viscoelastic lattice, crack can grow slowly (Slepyan *et al.*, 1999) if the relaxation and creep times belong to a static-amplitude-response domain, where the dynamic factor does not manifest itself, and vice versa. In particular, the crack cannot grow slowly in an elastic lattice (Slepyan, 1981a; Marder and Gross, 1995). *Instability of straight-line fast crack propagation* is shown (Marder and Gross, 1995). *Size effect in fracture* is revealed by the viscoelastic lattice model (Slepyan *et al.*, 1999; Slepyan, 1999).

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# Bi-modal surface energy and microcracking

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In classical fracture mechanics, equilibrium configurations are obtained by minimizing an energy functional containing two contributions, bulk and surface. Usually the bulk energy is convex and the surface energy is concave. While this type of minimization successfully describes macroscopic cracks, it has so far failed to model micro-defects forming a so-called process zone. In this paper we suggest a new model of brittle micro-damage with a non-concave, "bi-modal" surface energy, which allows the formation of both macro and micro cracks. The corresponding force-opening relation is characterized now by two peaks: the one near the origin and another one away from the origin. The important mathematical consequence of this assumption is that the surface energy is no longer subadditive which prevents the localization of fracture. Specifically, we consider the simplest one-dimensional problem for a bar in a hard device and show that as the total elongation increases, the model predicts the 'quantized" formation of a finite number of micro-cracks, one after another, until an ultimate macro-crack forms. The resulting total energy turns out to be a non-smooth function of average strain: it can be represented by a finite number of convex curves, each corresponding to a configuration with a fixed number of cracks. The overall stress-strain relation is then discontinuous and has a characteristic sawtooth structure. When the concave segment of the surface energy near the origin shrinks to zero the model recovers distributed damage, while when the convex region shrinks, a localized fracture appears as another limiting case. (Joint work with G. Del Piero).

# Dynamic perturbation of a crack propagating in a viscoelastic medium

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The problem is to find the stress intensity factors when a crack occupying the region

$$-\infty < x_1 < Vt, \ -\infty < x_2 < \infty, \ x_3 = 0$$

is subjected to general dynamic loading. The medium through which the crack propagates is viscoelastic (an elastic medium being included as a special case). The intensity factors are expressed as integrals over the crack surfaces of the applied loads, multiplied by kernel functions which are called weight functions. Equations which define the weight functions are set up. They yield Wiener-Hopf problems: a scalar problem for tensile loading, and a $2 \times 2$ matrix problem for shear loading. Surprisingly, both can be solved. The weight functions find use in calculating the perturbations to the stress intensity factors induced when the crack deviates slightly from being flat and straight-edged, and provide a tool for a stability analysis of the original crack, and for study of the development of "crack front disorder". The viscoelastic solution has not yet been published, but it builds upon ich is summarised in the references.

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